

A Continuous Increase in Lifetime, Uncertainty,
and the Dissaving Behavior of the Elderly*

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Abstract

This paper examines the relationship between a continuous increase in lifetime, uncertainty, and dissaving. It shows that an increase in lifetime accompanies uncertainty and this uncertainty negatively affects dissaving of the elderly. This result is consistent with the finding that the elderly do not dissave as fast as the life-cycle hypothesis predicts.

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1 Introduction

It is commonly said that savings decrease as the population ages. However, this is empirically hard to detect. For example, Japan's national saving rate didn't decrease in the 1980s and 1990s even though the ratio of the elderly steadily rose. At the individual level, various empirical studies also show that the elderly do not dissave as fast as the life-cycle hypothesis predicts.

Kageyama (2003) demonstrates that a greater increase in lifetime leads to greater savings. Theoretically, this is because of the following two reasons. At the individual level, the uncertainty associated with an increase in lifetime depresses dissaving. At the aggregate level, the working-age cohort whose lifetime is longer saves more than the retired cohort dissaves. This indicates that an aging population will not necessarily lead to a decrease in saving, depending on the source of an aging population. On one hand, an aging population caused by an increase in lifetime has a positive effect on saving while on the other, the aging of baby boomers has a negative effect. This hypothesis is tested empirically with cross-country data, and the results confirmed that an increase in life expectancy has a positive effect on various types of saving rates.

This paper explains how the uncertainty associated with an increase in lifetime depresses dissaving of the elderly. In the following two sections, I clarify problems of using the life-table data to estimate life expectancy and explain how an increase in lifetime influences the survival curve using the same theoretical framework as Kageyama (2003). Section IV shows the relationship between an increase in lifetime and dissaving. Concluding remarks are made in the final section.

2 Lifetime Increase and Life Tables

To analyze the effect of an increase in lifetime on dissaving, Kageyama (2003) follows the model in Yaari (1965). Suppose that the utility function is the CRRA type and that lifetime uncertainty is the only source of uncertainty in the future. Then, with $P(x, t, j)$ denoting the probability of surviving at least x more years for those born in year j and alive in year t (age $t - j$), the expected intertemporal utility function that an individual born in year

j and alive in year t faces becomes

$$E[V(c)] = \int_0^{\bar{X}(t,j)} P(x,t,j)e^{-\rho x} \frac{1}{1-\gamma} c(x,t,j)^{1-\gamma} dx, \quad 0 < x < \bar{X}(t,j) \quad (1)$$

where $\bar{X}(t,j)$ is the maximum remaining lifespan of cohort j in year t , $c(\cdot)$ is consumption, and $e^{-\rho x}$ is the subjective discount factor. Next, letting $W(\cdot)$, $y(x,t,j)$, and r represent net assets, the individual's earnings (other than interest), and the rate of interest, the change in net assets can be written as $\dot{W}(x,t,j) = y(x,t,j) - c(x,t,j) + rW(x,t,j)$. Then, the change in consumption is given by

$$\frac{\dot{c}(x,t,j)}{c(x,t,j)} = \frac{1}{\gamma} \left(r - \rho + \frac{\dot{P}(x,t,j)}{P(x,t,j)} \right) \quad (2)$$

when $W(x,t,j) > 0$. Subsequently, by integrating equation (2), the level of consumption at year t becomes

$$c(x,t,j) = [P(x,t,j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x} c(t,j) \quad (3)$$

where $c(t,j)$ is the consumption level at year t .

In order to focus on dissaving by the elderly, suppose that every individual retires at year t and receives no earnings other than interest after retirement. Thus, the elderly dissave from their assets accumulated during working years. With this simplification and the terminal condition that $W[\bar{X}(t,j)] = 0$, the optimal consumption level at year t becomes

$$c(t,j) = \frac{W(t,j)}{\int_0^{\bar{X}(t,j)} [P(x,t,j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x-rx} dx} \quad (4)$$

where $W(t,j)$ is the level of assets accumulated by year t .

It is obvious from equation (4) that the level of consumption, or dissaving, largely depends on the shape of the survival curve, $P(x,t,j)$ for $0 \leq x \leq \bar{X}(t,j)$. However, the true $P(x,t,j)$ is unveiled only in the future. This means that people use an estimated $P(x,t,j)$, not the true $P(x,t,j)$, to determine their level of consumption. Although it is common to assume that people apply the life table data to estimate the true $P(x,t,j)$, Kageyama (2003) shows that this life-table estimator, say $P_{LT}(x,t,j)$, is not a good estimator. Intuitively, this is because it does not incorporate the delay in physiological aging which anyone across all ages will experience. Due to

technological progress in health and medicine and improvements in general living conditions, the physiological process of aging is steadily slower than actual aging as measured in calendar years. Figure 1 shows the significance of an increase in lifetime. The dotted lines which are constructed from the 1962 life tables correspond to the life-table survival curves of those aged 60 in 1962. The solid lines represent the realized survival curves of the same cohort. Obviously, the life-table survival curves underestimate the realized survival curves for both sexes. This indicates that the physiological process of aging is expected to slow over time, which would correspond to an increase in lifetime.

In life tables, however, it is assumed that the age-specific mortality rate is constant across time. Under this assumption, $P_{LT}(x, t, j)$ can be calculated simply using the observed age-specific mortality rates of the older generations. These rates become proxies for the future age-specific mortality rates of the younger generations.

Practically, this approach starts by letting the realized age-specific mortality rate of cohort $j + i$ in the previous year be $m(t - 1, j + i)$ and standardizing the number of newborn children in year t to 100,000. Then, the hypothetical number of survivors in cohort j out of 100,000, $N(t, j)$, is calculated as $100,000 \prod_{i=0}^{t-j-1} [1 - m(t - 1, j + i)]$. In life tables, this hypothetical number of survivors is called the stationary population of cohort j in year t . The stationary population of cohort $j - x$ in year t , or x -year-older cohort, $N(t, j - x)$, can also be calculated using this method. Then, under the given assumption, the ratio of the stationary population of cohort $j - x$ to the one of cohort j , $\frac{N(t, j-x)}{N(t, j)}$, becomes the survival probability for cohort j applicable to x years later, which is $P_{LT}(x, t, j)$.

In order to incorporate a delay in aging, Kageyama (2003) introduced a new indicator, say $P_{DA}(y, t, j)$. The construction of this indicator starts with letting Y be the random variable expressing the number of years until death when an increase in lifetime is incorporated and Z (in years) the random variable denoting the delay in aging which occurs in one year.¹ Here, Y has the same support as X (the random variable expressing the number of years until death under life tables), and Z takes a value between a and b ($b < 1$)

¹In fact, there exists another random variable: white noise. This white noise represents the estimation error which exists even if the expected increase in lifetime is nil. However this estimation error is omitted since incorporating it does not change the result in any way.

with the density $f(z)$ and $E[Z] = \bar{z} > 0$. It is worth noting that Z represents the length of the delay in physiological aging, not the length of the increase in remaining life expectancy. Next, assume that Z is independent of both time and age. This implies that an individual in cohort j experiences a zx -year delay in physiological aging over the next x years. Therefore, $X + ZX$ yields the same distribution as Y .

Applying this result to the concept of stationary population, the hypothetical number of those alive x years later in cohort j is now given by $N(t, j - x + zx)$, the stationary population of cohort $j - x + zx$ in year t . In other words, the physiological age of cohort j after x years will be $t - j + x - zx$ whereas actual age will be $t - j + x$. Therefore, $P_{DA}(y, t, j)$ becomes $\frac{N(t, j - x + zx)}{N(t, j)}$.

This shows that the life-table estimator underestimates the survival probability by zx years over the next x years. In other words, remaining life expectancy is biased downwards by $E[Z]E[X]$ years in life tables.

3 Effect of Uncertainty

The discussion above shows that the life-table life expectancy is not appropriate to estimate lifetime since it does not include an increase in lifetime. However, the effect of an increase in lifetime is not limited to the increase itself, the uncertainty associated with the increase has its own effect. As shown in Figure 2, the rise in life expectancy is not necessarily smooth and this variation has its own influence on the estimated survival curve.² To examine the effect of this variation, ignore the uncertainty associated with Z for a moment and let $P_{NU}(y, t, j)$ be equal to $\frac{N(t, j - x + \bar{z}x)}{N(t, j)}$. Thus, P_{NU} can be considered as P_{DA} without the uncertainty attached to the increase, or alternatively it is expressed by $(1 + \bar{z})X$ in terms of a random variable. In this case, an increase in lifetime is assumed to be smooth, not as in Figure 2. Comparing P_{DA} with P_{NU} , the effect on the survival probabilities of the uncertainty attached to a lifetime increase becomes clear.

For this purpose, I use the technique in Rothschild and Stiglitz (1970). Following their study, start with defining a concave function v . With this v and Jensen's inequality, taking expectations with respect to Z yields the

²The increase in life expectancy appears smooth for several countries. This is due to data limitations, as most of the countries do not publish life expectancy on an annual basis.

relationship $E_X [v(X + ZX)] \leq v(X + \bar{z}X)$. Then, taking expectations again with respect to X , the relationship $EE_X [v(X + ZX)] \leq E [v(X + \bar{z}X)]$ is obtained. This relationship can be rewritten as

$$\int_0^{\bar{X}(t,j)} v(x) dS(x) \geq 0 \quad (5)$$

where $S(x) = P_{DA}(x, t, j) - P_{NU}(x, t, j)$.

Next, let $w_y(x) = \max(y - x, 0)$ for fixed y . Since $-w_y(x)$ is concave, the relationship $\int_0^{\bar{X}(t,j)} w_y(x) dS(x) = \int_0^y (y - x) dS(x) \leq 0$ must hold. Using the integral by parts, this is equivalent to

$$\int_0^y [P_{DA}(x, t, j) - P_{NU}(x, t, j)] dx \leq 0. \quad (6)$$

This means that the area under P_{NU} is at least as large as the area under P_{DA} for any y and larger at least one y since $P_{DA} \neq P_{NU}$.³ Then, Levhari and Mirman (1977) found that inequality (6) implies

$$\int_0^{\bar{X}(t,j)} [P_{DA}(y, t, j)]^{\frac{1}{\gamma}} dy \geq \int_0^{\bar{X}(t,j)} [P_{NU}(y, t, j)]^{\frac{1}{\gamma}} dy, \quad \text{for all } \gamma > 1. \quad (7)$$

This clarifies the relationship between P_{DA} and P_{NU} .

As an example, assume X and $1 + Z$ follow uniform distributions with interval $(0, A)$ and $(0, B)$ respectively.⁴ With this assumption, the density of Y is given by

$$\begin{aligned} q_Y(y) &= \int_{-\infty}^{\infty} \frac{1}{x} \frac{1}{A} I_{(0,A)}(x) \frac{1}{B} I_{(0,B)}\left(\frac{y}{x}\right) dx \\ &= \frac{1}{AB} I_{(0,AB)}(y) \int_0^{AB} \frac{1}{x} I_{(\frac{y}{B}, A)}(x) dx \\ &= \frac{1}{AB} I_{(0,AB)}(y) \int_{\frac{y}{B}}^A \frac{1}{x} dx \\ &= \frac{1}{AB} \log\left(\frac{AB}{y}\right) I_{(0,AB)}(y) \end{aligned} \quad (8)$$

where $I_{(\alpha,\beta)}(\omega)$ is the indicator function with the value equal to 1 if ω is within the interval (α, β) and 0 otherwise. Then, the cumulative density

³They are equal at $y = \bar{X}(t, j)$.

⁴Though the survival curve is normally expressed by Weibull distribution, I use uniform distributions for simplicity.

becomes

$$\begin{aligned} Q_Y(y) &= I_{(0,AB)}(y) \int_0^y \frac{1}{AB} \log\left(\frac{AB}{x}\right) dx \\ &= \frac{1}{AB} \left[y + y \log\left(\frac{AB}{y}\right) \right] I_{(0,AB)}(y) \end{aligned}$$

and $P_{DA}(y, t, j)$, which is equal to $1 - Q_Y(y)$, is given by

$$P_{DA}(y, t, j) = 1 - \frac{1}{AB} \left[y + y \log\left(\frac{AB}{y}\right) \right] I_{(0,AB)}(y). \quad (9)$$

On the other hand, the probability based on $(1 + \bar{z})X$, $P_{NU}(y, t, j)$, becomes

$$P_{NU}(y, t, j) = 1 - Q_{\frac{B}{2}X}(y) = 1 - \frac{2y}{AB} I_{(0, \frac{AB}{2})}(y) \quad (10)$$

since $(1 + \bar{z}) = \frac{B}{2}$.

Of course, both distributions give the same mean, $\frac{AB}{4}$. However, the shape of the survival curve is not the same. Except for the terminal points, equation (9) and (10) yield the same value at only one point, $y = AB e^{-1}$, and the slopes of the curves at this point are respectively $-\frac{1}{AB}$ and $-\frac{2}{AB}$ for P_{DA} and P_{NU} . This indicates that P_{DA} takes the smaller values until it reaches $AB e^{-1}$ and the larger values after it passes $AB e^{-1}$. This satisfies the inequality (6). Figure 3 gives the visual presentation of the relationship between P_{DA} and P_{NU} .

4 Effects on Dissaving

In the previous sections, we examined how the survival probabilities are affected by an increase in lifetime. The relationship between P_{LT} and P_{DA} is presented in Figure 4. The solid and thick lines represent the life-table survival curve and the survival curve under P_{DA} . The dotted line indicates the survival curve under P_{NU} . It shows that a continuous increase in lifetime makes the survival curve higher and flatter. This result is consistent with the finding in Hamermesh (1985) that the life-table survival curve second-order stochastically dominates the personal expected survival curve.

These results together with equation (4) indicates that, as an increase in lifetime gets larger, the level of dissaving becomes lower than we formerly thought with the life-table estimator. This is because of the following two

reasons. First, the life-table estimator underestimates the survival probabilities since it can not incorporate an increase in lifetime. Therefore, a greater increase in lifetime leads to higher $P(x, t, j)$ for any x and this relationship is written as

$$\int_0^{\bar{X}(t,j)} [P_{NU}(x, t, j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x-rx} dx \geq \int_0^{\bar{X}(t,j)} [P_{LT}(x, t, j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x-rx} dx. \quad (11)$$

The second effect, the uncertainty attached to the increase, also has a negative effect on dissaving. In theory, this depends on the values of r , ρ , and especially γ as in Levhari and Mirman (1977). For example, γ needs to be larger than unity when r and ρ are negligible. Thus, the effect of the lifetime uncertainty on savings is an empirical matter and Davies (1981) found that the inequality

$$\int_0^{\bar{X}(t,j)} [P_{DA}(x, t, j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x-rx} dx \geq \int_0^{\bar{X}(t,j)} [P_{NU}(x, t, j)]^{\frac{1}{\gamma}} e^{\frac{1}{\gamma}(r-\rho)x-rx} dx \quad (12)$$

holds with appropriate estimates of these variables. Incorporating this result in equation (4), people are expected to reduce dissaving when the uncertainty with respect to lifetime exists.

5 Conclusion

In this paper, I pointed out the importance of an increase in lifetime to explain the dissaving behavior of the elderly. This result goes with empirical findings in Kageyama (2003) that the increase in lifetime is found significant to explain various saving rates. The increase in lifetime is particularly significant among developed countries and the high saving rate of Japan can mainly be attributed to its large increase in lifetime.

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Figure 1: Comparison between Survival Curves

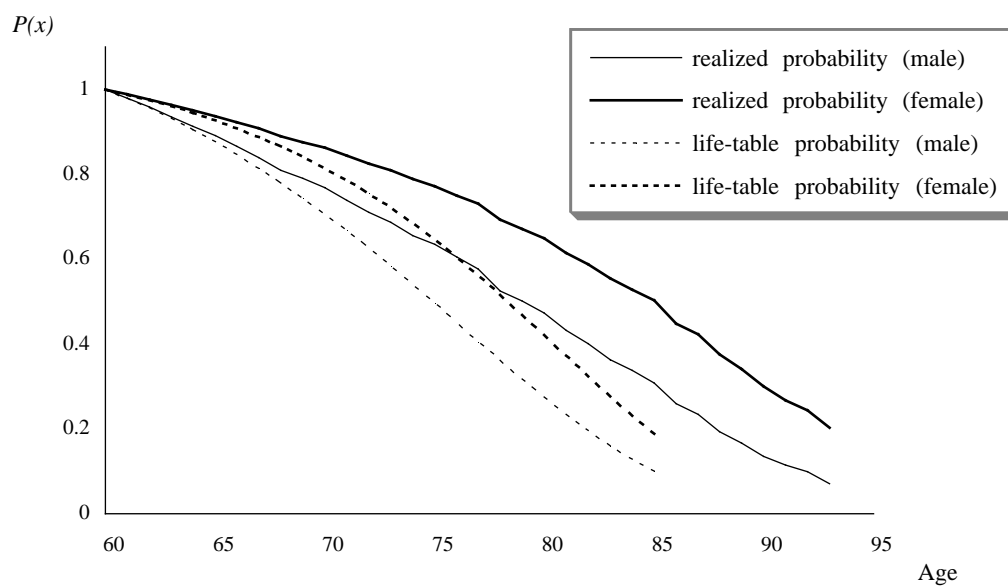
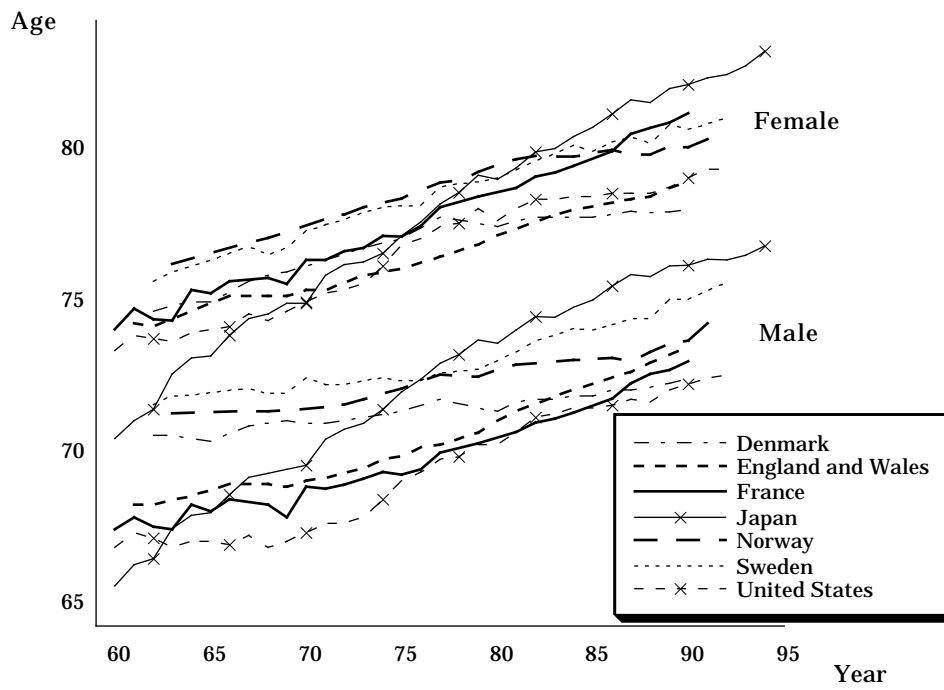


Figure 2: Changes in Life Expectancy



Source: Ministry of Health and Welfare of Japan (1998), 18th Life Tables

Figure 3: Effect of Uncertainty on Survival Curve

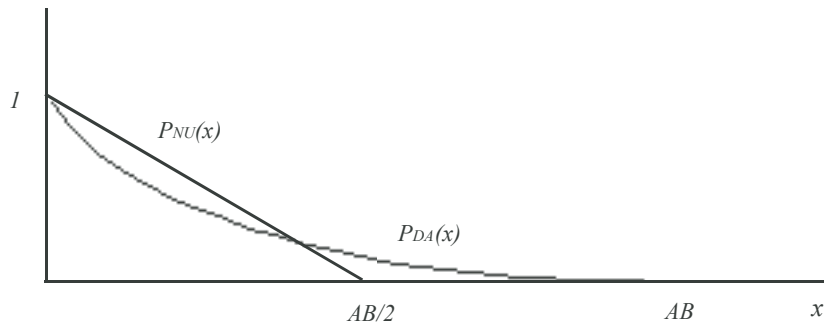


Figure 4: Effect of a Continuous Increase in Lifetime on Survival Curve

